IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 04, Issue 09 (September. 2014), ||V4|| PP 32-37

A Quarter-Symmetric Non-Metric Connection In A Lorentzian $\boldsymbol{\beta}$ −Kenmotsu Manifold

Shyam Kishor[1], Prerna Kanaujia[2], Satya Prakash Yadav[3]
[1&2]Department of Mathematics & Astronomy University of Lucknow-226007, India , [3]SIET, Allahabad

Abstract:- In this paper we study quarter- symmetric non-metric connection in a Lorentzian $\beta-$ kenmotsu manifold and the first Bianchi identity for the curvature tensor is found. Ricci tensor and the scalar curvature with respect to quarter-symmetric non-metric connection in Lorentzian β −Kenmotsu manifold are obtained. Finally some identities for torsion tensor have been explored.

Keywords- *Hayden connection, Levi-Civita connection, Lorentzian β −kenmotsu manifold, quarter- symmetric metric connection, quarter symmetric non-metric connection.*
MSC 2010: *53B20, 53B15, 53C15.*

1. INTRODUCTION

Let M be an n −dimensional differentiable manifold equipped with a linear connection $\widetilde{\nabla}$. The torsion tensor $\tilde{T}$ of $\widetilde{\nabla}$ is given by

$$T(X,Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X,Y]$$

$$R(X,Y)Z = \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y\}} Z.$$

The connection $\widetilde{\nabla}$ is symmetric if its torsion tensor $\tilde{T}$ vanishes, otherwise it is non-symmetric. If there is a Riemannian metric g in M such that $\tilde{\nabla} g = 0$, the connection $\widetilde{\nabla}$ is a metric connection, otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection.

Hayden [24] introduced a metric connection $\widetilde{\nabla}$ with non-zero torsion on a Riemannian manifold. Such a connection is called a Hayden connection. On the other hand, in a Riemannian manifold given a 1 −form ω, the Weyl connection $\widetilde{\nabla}$ constructed with ω and its associated vector B (Folland 1970, [1]) is a symmetric non-metric connection. In fact, the Riemannian metric of the manifold is recurrent with respect to the Weyl connection with the recurrence 1 −form ω, that is,$\widetilde{\nabla} g = \omega \otimes g$. Another symmetric non-metric connection is projectively related to the Levi-Civita connection (cf. Yano [19], Smaranda [25]). Friedmann and Schouten ([2], [20]) introduced the idea of a semi-symmetric linear connection in a differentiable manifold. A linear connection is said to be a semi-symmetric connection if its torsion tensor $\tilde{T}$ is of the form

$$\tilde{T}(X,Y) = u(Y)X - u(X)Y \qquad (1.1)$$

where u is a 1 −form. A Hayden connection with the torsion tensor of the form (1.1) is a semi-symmetric metric connection. In 1970, Yano [3] considered a semi-symmetric metric connection and studied some of its properties. Some different kinds of semi-symmetric connections are studied in [4], [5], [6] and [7].In 1975, S. Golab [8] defined and studied quarter-symmetric linear connections in differentiable manifolds. A linear connection is said to be a quarter-symmetric connection if its torsion tensor $\tilde{T}$ is of the form

$$\tilde{T}(X,Y) = u(Y)\varphi X - u(X)\varphi Y \qquad ,X,Y \in TM \qquad (1.2)$$

where u is a 1 −form and φ is a tensor of type $(1,1)$. Note that a quarter-symmetric metric connection is a Hayden connection with the torsion tensor of the form (1.2). Studies of various types of quarter-symmetric metric connections and their properties include [9], [10], [11] and [12] among others.

On the other hand, there is well known class of almost contact metric manifolds introduced by K. Kenmotsu, which is now known as Kenmotsu manifolds [10]. An almost contact metric structure on a manifold M is called a trans-Sasakian structure if the product manifold $M \times R$ belongs to the class $W4$. The class $C_6 \oplus C_5$ ([13], [26]) coincides with the class of the trans-Sasakian structures of type (α,β). In fact, in [13], local nature of the two subclasses, namely, C_5 and C_6 structures of trans-Sasakian structures are characterized completely.
We note that trans-Sasakian structures of type $(0,0)$, $(0,\beta)$ and $(\alpha,0)$ are cosymplectic [21], β−Kenmotsu [14] and α −Sasakian [14] respectively. The paper is organized as follows:

Section 2, deals with some preliminary results about quarter-symmetric non-metric connection. In this section the curvature tensor of the Riemannian manifold with respect to the defined quarter-symmetric non-metric connection is also found. In the last of this section first Bianchi identity for the curvature tensor of the Riemannian manifold with respect to the given quarter-symmetric non-metric connection is found. In section 3, we study this quarter-symmetric non-metric connection in Lorentzian β−Kenmotsu manifold. We have given the covariant derivative of a $1-$form and the torsion tensor. We also get the curvature tensor of the Lorentzian $\beta-$Kenmotsu manifold with respect to the defined quarter-symmetric non-metric connection and find first Bianchi identity. Finally we have calculated Ricci tensor, scalar curvature and torsion tensor of the Lorentzian $\beta-$ Kenmotsu manifold with respect to the defined quarter-symmetric non-metric connection.

2. A Quarter-Symmetric Connection

In this section existence of quarter-symmetric non-metric connection has been discussed.

Theorem-1 Let M be an n-dimensional Riemannian manifold equipped with the Levi-Civita connection $\tilde{\nabla}$ of its Riemannian metric g. Let η be a 1-form and φ, a (1,1) tensor field in M such that

$$\eta(X) = g(\xi, X), \tag{2.1}$$

$$g(\varphi X, Y) = -g(X, \varphi Y) \tag{2.2}$$

for all $X, Y \in TM$. Then there exists a unique quarter- symmetric non-metric connection $\tilde{\nabla}$ in M given by

$$\tilde{\nabla}_X Y = \nabla_X Y - \eta(X)\varphi Y - g(X, Y)\xi, \tag{2.3}$$

That satisfies

$$\tilde{T}(X, Y) = \eta(Y)\varphi X - \eta(X)\varphi Y, \tag{2.4}$$

and

$$\tilde{\nabla}_X g(Y, Z) = \eta(Y)g(X, Z) + \eta(Z)g(X, Y) \tag{2.5}$$

where $\tilde{T}$ is the torsion tensor of $\tilde{\nabla}$.

Proof: The equation (2.4) of [15] is

$$\begin{aligned}\tilde{\nabla}_X Y &= \nabla_X Y + u(Y)\varphi_1 X - u(X)\varphi_2 Y - g(\varphi_1 X, Y)U \\ &\quad - f_1\{u_1(X)Y + u_1(Y)X - g(X, Y)U_1\} - f_2 g(X, Y)U_2\end{aligned}$$

Taking

$$\varphi_1 = 0, \varphi_2 = \varphi, u = u_1 = \eta, f_1 = 0, f_2 = 1, U_2 = \xi, \tag{2.6}$$

in above equation, we get (2.3). The equations (2.5) and (2.6) of [15] are

$$\tilde{T}(X, Y) = u(Y)\varphi X - u(X)\varphi Y,$$

$$\tilde{\nabla}_X g(Y, Z) = 2f_1 u_1(X)g(Y, Z) + f_2\{u_2(Y)g(X, Z) + u_2(Z)g(X, Y)\}$$

Using (2.6) in above equations, we get respectively (2.4) and (2.5).

Conversely, a connection defined by (2.3) satisfies the condition (2.4) and (2.5).

Proposition 1. Let M be an n-dimensional Riemannian manifold. For the quarter-symmetric connection defined by (2.3), the covariant derivatives of the torsion tensor $\tilde{T}$ and any 1-form π are given respectively by

$$\begin{aligned}(\tilde{\nabla}_X \tilde{T})(Y, Z) &= ((\tilde{\nabla}_X \eta)Z)\varphi Y - (\tilde{\nabla}_X \eta)Y)\varphi Z \\ &\quad + \eta(Z)(\tilde{\nabla}_X \varphi)Y - \eta(Y)(\tilde{\nabla}_X \varphi)Z,\end{aligned} \tag{2.7}$$

and

$$(\tilde{\nabla}_X \pi)Y = (\nabla_X \pi)Y + \eta(X)\pi(\varphi Y) + g(X, Y)\pi(\xi) \tag{2.8}$$

for all $X, Y, Z \in T M$.

Using (2.8) & (2.3) in

$$(\tilde{\nabla}_X \tilde{T})(Y, Z) = \tilde{\nabla}_X \tilde{T}(Y, Z) - \tilde{T}(\tilde{\nabla}_X Y, Z) - \tilde{T}(Y, \tilde{\nabla}_X Z)$$

We obtain (2.7). Similarly, using (2.3) with

$$(\tilde{\nabla}_X \pi)Y = \tilde{\nabla}_X \pi Y - \pi(\tilde{\nabla}_X Y)$$

(2.8) can be obtained.

In an n-dimensional Riemannian manifold M, for the quarter-symmetric connection defined by (2.3), let us write

$$\tilde{T}(X, Y, Z) = g(\tilde{T}(X, Y), Z), \qquad X, Y, Z \epsilon TM. \tag{2.9}$$

Proposition 2. Let M be an n-dimensional Riemannian manifold. Then

$$\begin{aligned}&\tilde{T}(X, Y, Z) + \tilde{T}(Y, Z, X) + \tilde{T}(Z, X, Y) \\ &= 2\eta(X)g(Y, \varphi Z) + 2\eta(Y)g(Z, \varphi X) + 2\eta(Z)g(X, \varphi Y)\end{aligned} \tag{2.10}$$

Proof: In view of (2.7) and (2.9) we have the proposition.

Theorem 2. Let M be an n-dimensional Riemannian manifold equipped with the Levi-Civita connection $\tilde{\nabla}$ of its Riemannian metric g. Then the curvature tensor $\tilde{R}$ of the quarter-symmetric connection defined by (2.3) is given by

$$\tilde{R}(X, Y, Z) = R(X, Y, Z) - \tilde{T}(X, Y, Z)\xi - 2d\eta(X, Y)\varphi Z$$

$$+\eta(X)(\nabla_Y\varphi)Z-\eta(Y)(\nabla_X\varphi)Z$$
$$+g(Y,Z)\{\eta(X)\xi-\nabla_X\xi+\eta(X)\varphi\xi\}$$
$$-g(X,Z)\{\eta(Y)\xi-\nabla_Y\xi+\eta(Y)\varphi\xi\} \qquad (2.11)$$

for all $X,Y,Z\in TM$, where R is the curvature of Levi-Civita connection.

Proof: In view of (2.3), (2.2), (2.4) and (2.9) we get (2.11).

Theorem 3. In an n-dimensional Riemannian manifold the first Bianchi identity for the curvature tensor of the Riemannian manifold with respect to the quarter-symmetric connection defined by (2.3) is

$$\tilde{R}(X,Y,Z)+\tilde{R}(Y,Z,X)+\tilde{R}(Z,X,Y)$$
$$=-\{\tilde{T}(X,Y,Z)\xi+\tilde{T}(Y,Z,X)\xi+\tilde{T}(Z,X,Y)\xi\}$$
$$+\eta(X)B(Y,Z)+\eta(Y)B(Z,X)+\eta(Z)B(X,Y)$$
$$-2d\eta(X,Y)\varphi Z-2d\eta(Y,Z)\varphi X-2d\eta(Z,X)\varphi Y \qquad (2.12)$$

for all $X,Y,Z\in TM$, where

$$B(X,Y)=(\nabla_X\varphi)Y-(\nabla_Y\varphi)X. \qquad (2.13)$$

Proof: From (2.11), we get

$$\tilde{R}(X,Y,Z)+\tilde{R}(Y,Z,X)+\tilde{R}(Z,X,Y)$$
$$=2\eta(X)g(\varphi Y,Z)\xi+2\eta(Y)g(\varphi Z,X)\xi+2\eta(Z)g(\varphi X,Y)\xi$$
$$+\eta(X)(\nabla_Y\varphi)Z-\eta(X)(\nabla_Z\varphi)Y+\eta(Y)(\nabla_Z\varphi)X$$
$$-\eta(Y)(\nabla_X\varphi)Z+\eta(Z)(\nabla_X\varphi)Y-\eta(Z)(\nabla_Y\varphi)X$$
$$-\big((\nabla_X\eta)Y\big)\varphi Z+\big((\nabla_Y\eta)X\big)\varphi Z-\big((\nabla_Y\eta)Z\big)\varphi X$$
$$+\big((\nabla_Z\eta)Y\big)\varphi X-\big((\nabla_Z\eta)X\big)\varphi Y+\big((\nabla_X\eta)Z\big)\varphi Y.$$

Using (2.13) and (2.10) in the previous equation we get (2.12).

Let us write the curvature tensor $\tilde{T}$ as a (0,4) tensor by

$$\tilde{R}(X,Y,Z,W)=g\big(\tilde{R}(X,Y)Z,W\big), \quad X,Y,Z,W\in TM. \qquad (2.14)$$

Then we have the following:

Theorem 4. Let M be a Riemannian manifold. Then

$$\tilde{R}(X,Y,Z,W)+\tilde{R}(Y,X,Z,W)=0, \qquad (2.15)$$

for all $X,Y,Z,W\in TM$.

Proof: Using (3.25) in (2.14), we get

$$\tilde{R}(X,Y,Z,W)=R(X,Y,Z,W)-\eta(Z)g\big(\tilde{T}(X,Y),W\big)$$
$$+g(Y,Z)g(X,W)-g(X,Z)g(Y,W). \qquad (2.16)$$

Interchanging X and Y in the previous equation and adding the resultant equation in (3.28) and using (2.4) we get (2.15).

3. QUARTER-SYMMETRIC NON-METRIC CONNECTION IN A LORENTZIAN β −KENMOTSU MANIFOLD

A differentiable manifold M of dimension n is called Lorentzian Kenmotsu manifold if it admits a (1,1) −tensor φ, a contravariant vector field ξ, a covariant vector field η and Lorentzian metric g which satisfy

$$\varphi^2X=X+\eta(X)\xi, \quad \eta(\xi)=-1, \quad \varphi\xi=0, \qquad \eta(\varphi X)=0 \qquad (3.1)$$
$$g(\varphi X,\varphi Y)=g(X,Y)+\eta(X)\eta(Y) \qquad (3.2)$$
$$g(\varphi X,Y)=g(X,\varphi Y), \qquad g(X,\xi)=\eta(X) \qquad (3.3)$$

for all $X,Y\in TM$.

Also if Lorentzian Kenmotsu manifold M satisfies

$$(\nabla_X\varphi)(Y)=\beta[g(\varphi X,Y)\xi-\eta(Y)\varphi X], \quad X,Y\in TM \qquad (3.4)$$

Where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g, then M is called Lorentzian β-Kenmotsu manifold. From the above equation it follows that

$$\nabla_X\xi=\beta[X-\eta(X)\xi] \qquad (3.5)$$

$$(\nabla_X\eta)(Y)=\beta[g(X,Y)-\eta(X)\eta(Y)] \qquad (3.6)$$

and consequently

$$d\eta=0 \qquad (3.7)$$

Where

$$d\eta(X,Y) = \frac{1}{2}((\nabla_X\eta)(Y) - (\nabla_Y\eta)(X), \quad X,Y \in TM \tag{3.8}$$

Furthur, on a Lorentzian β-Kenmotsu manifold M, the following relations hold ([3], [16])

$$\eta(R(X,Y)Z) = \beta^2[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)] \tag{3.9}$$

$$R(X,Y)\xi = \beta^2[\eta(X)Y - \eta(Y)X] \tag{3.10}$$

$$S(X,\xi) = -(n-1)\beta^2\eta(X) \tag{3.11}$$

$$Q\xi = -(n-1)\beta^2\xi \tag{3.12}$$

$$S(\xi,\xi) = (n-1)\beta^2 \tag{3.13}$$

The equation (3.9) is equivalent to

$$R(\xi,X)Y = \beta^2[\eta(Y)X - g(X,Y)\xi] \tag{3.14}$$

which implies that

$$R(\xi,X)\xi = \beta^2[X - \eta(X)\xi] \tag{3.15}$$

From (3.9) and (3.14), we have

$$\eta(R(X,Y)\xi) = 0 \tag{3.16}$$

$$\eta(R(\xi,X)Y) = \beta^2[\eta(Y)\eta(X) - g(X,Y)]. \tag{3.17}$$

Theorem 5. Let M be a Lorentzian β−Kenmotsu manifold. Then for the quarter-symmetric connection defined by (2.3), we have

$$(\tilde{\nabla}_X\pi)Y = (\nabla_X\pi)Y + \eta(X)\pi(\varphi Y) + g(X,Y)\pi(\xi). \tag{3.18}$$

In particular**,**

$$(\tilde{\nabla}_X\eta)Y = (\beta-1)g(X,Y) - \beta\eta(X)\eta(Y) \tag{3.19}$$

and

$$\tilde{d}\eta = 0 \tag{3.20}$$

where

$$\tilde{d}\eta(X,Y) = \frac{1}{2}[(\tilde{\nabla}_X\eta)(Y) - (\tilde{\nabla}_Y\eta)(X)], \quad X,Y \in TM \tag{3.21}$$

Proof: From equation (2.3), we get (3.18). Now replacing π by η in (3.18) and using (3.1) and (3.6) we get (3.19). Equation (3.20) follows immediately from (3.19).

Theorem 6. Let M be a Lorentzian β-Kenmotsu manifold. Then

$$(\tilde{\nabla}_X\varphi)Y = (\beta+1)g(\varphi X,Y)\xi - \beta\eta(Y)\varphi X \tag{3.22}$$

which implies

$$\tilde{T}(X,Y) = \frac{1}{\beta}[-(\tilde{\nabla}_X\varphi)Y + (\tilde{\nabla}_Y\varphi)X] + \frac{(\beta+1)}{\beta}[g(\varphi X,Y)\xi - g(\varphi Y,X)\xi] \tag{3.23}$$

and

$$\tilde{\nabla}_X\xi = \beta X - (\beta+1)\eta(X)\xi \tag{3.24}$$

For all $X,Y \in TM$.

Proof: From Equations (2.3) and (3.1), we get

$$(\tilde{\nabla}_X\varphi)Y = (\nabla_X\varphi)Y + g(X,\varphi Y)\xi,$$

Which in view of (3.4) gives (3.22). From (3.22) and (2.4), we get (3.23). Now in view of equations (2.3), (3.5), (3.1) and (3.3), we get (3.24).

Theorem 7. Let M be a Lorentzian β-Kenmotsu manifold. Then

$$\tilde{\nabla}_X\tilde{T}(Y,Z) = (\beta+1)[g(X,Z)\varphi Y - g(X,Y)\varphi Z + \eta(Z)g(\varphi X,Y)\xi - \eta(Y)g(\varphi X,Z)\xi] - \beta\eta(X)\tilde{T}(Y,Z) \tag{3.25}$$

Consequently,

$$\tilde{\nabla}_X\tilde{T}(Y,Z) + \tilde{\nabla}_Y\tilde{T}(Z,X) + \tilde{\nabla}_Z\tilde{T}(X,Y) = 0 \tag{3.26}$$

for all $X,Y,Z,W \in TM$.

Proof: Using equations (3.19), (3.22) and (2.4) in (2.7) we obtain (3.25). Equation (3.26) follows from (3.25) and (2.4).

Theorem 8. The curvature tensor $\tilde{R}$ of the quarter-symmetric connection in a Lorentzian β-kenmotsu manifold is as follows

$$\tilde{R}(X,Y)Z = R(X,Y)Z + (\beta+1)[\eta(X)g(\varphi Y,Z)\xi$$

$$-\eta(Y)g(\varphi X,Z)\xi+g(Y,Z)\eta(X)\xi-g(X,Z)\eta(Y)\xi]$$
$$+\beta[\eta(Z)\tilde{T}(X,Y)-g(Y,Z)X+g(X,Z)Y] \qquad (3.27)$$

Proof: Using (3.1), (3.3), (2.9) and (2.4) in (2.11), we get

$$\begin{aligned}\tilde{R}(X,Y)Z=&R(X,Y)Z-\eta(Y)(\nabla_X\varphi)Z+\eta(X)(\nabla_Y\varphi)Z\\&+((\nabla_Y\eta)X)\varphi Z-((\nabla_X\eta)Y)\varphi Z\\&+g(Y,Z)(-\nabla_X\xi+\eta(X)\xi)\\&-g(X,Z)(-\nabla_Y\xi+\eta(Y)\xi)\\&-\eta(Y)g(\varphi X,Z)\xi+-\eta(X)g(\varphi Y,Z)\xi.\end{aligned}$$

Now using (3.4), (3.7), (3.5) and (3.1) in the above equation we obtain (3.27).

Now for the curvature tensor $\tilde{R}$ of the quarter-symmetric non-metric connection of the Lorentzian β-Kenmotsu manifold we have following theorems.

Theorem 9. Let M be a Lorentzian β-Kenmotsu manifold. Then

$$\begin{aligned}&\tilde{R}(X,Y,Z,W)+\tilde{R}(X,Y,W,Z)\\&=\beta\left[\eta(Z)g\left(\tilde{T}(X,Y),W\right)+\eta(W)g\left(\tilde{T}(X,Y),Z\right)\right]\\&+(\beta+1)[\{\eta(X)g(\varphi Y,Z)-\eta(Y)g(\varphi X,Z)\\&+g(Y,Z)\eta(X)-g(X,Z)\eta(Y)\}\eta(W)\\&+\{\eta(X)g(\varphi Y,W)-\eta(Y)g(\varphi X,W)\\&+g(Y,W)\eta(X)-g(X,W)\eta(Y)\}\eta(Z)] \qquad (3.28)\end{aligned}$$

and

$$\begin{aligned}&\tilde{R}(X,Y,Z,W)-\tilde{R}(Z,W,X,Y)\\&=\beta\left[\eta(Z)g\left(\tilde{T}(X,Y),W\right)-\eta(X)g\left(\tilde{T}(Z,W),Y\right)\right]\\&+(\beta+1)[\eta(X)\eta(W)\{g(\varphi Y,Z)+g(Y,Z)\}\\&-\eta(Y)\eta(Z)\{g(\varphi W,X)+g(W,X)\}] \qquad (3.29)\end{aligned}$$

for all $X,Y,Z,W\in TM$.

Proof: From (3.27) equation (2.14) reduces to

$$\begin{aligned}\tilde{R}(X,Y,Z,W)=&R(X,Y,Z,W)+\beta[\eta(Z)g\left(\tilde{T}(X,Y),W\right)\\&-g(Y,Z)g(X,W)+g(X,Z)g(Y,W)]\\&+(\beta+1)[\eta(W)\{\eta(X)g(\varphi Y,Z)-\eta(Y)g(\varphi X,Z)\\&+g(Y,Z)\eta(X)-g(X,Z)\eta(Y)\}] \qquad (3.30)\end{aligned}$$

Interchanging X and Y in the above equation and adding the resultant equation in it and then using (2.4) we get (2.15). Now interchanging Z and W in (3.30) and adding the resultant equation to (3.30) we obtain (3.28). In the last the equation (3.29) can be obtained by interchanging X and Z & Y and W in (3.30) and substracting the resultant equation from (3.30) and using (2.4).

Theorem 10: The first Bianchi identity for the curvature tensor of the Lorentzian β −Kenmotsu manifold with respect to the connection defined in the equation (2.3) is as given below

$$\tilde{R}(X,Y)Z+\tilde{R}(Y,Z)X+\tilde{R}(Z,X)Y=0 \qquad (3.31)$$

for all $X,Y,Z\in TM$.

Proof: In view of equations (3.27) and (2.4), we get (3.31).

Theorem 11. In an n-dimensional Lorentzian β-Kenmotsu manifold M, the Ricci tensor and the scalar curvature with respect to the connection defined by the equation (2.3) are given by

$$\begin{aligned}\tilde{S}(Y,Z)=&S(Y,Z)-\beta(n-1)g(Y,Z)\\&+(\beta+1)\begin{bmatrix}g(\varphi Y,Z)-\eta(Y)g(\varphi Z,\xi)\\-\eta(Y)\eta(Z)\end{bmatrix} \qquad (3.32)\end{aligned}$$

where $X,Y\in TM$ and

$$\tilde{r}=r-\beta n(n-1)-(\beta+1) \qquad (3.33)$$

respectively. Where S is the Ricci tensor and r is the scalar curvature of M.

Proof: Let $\{e_1, e_2,\ldots\ldots, e_n\}$ be a basis of M, then

$$S(Y,Z)=\sum g(\tilde{R}(e_i,Y)Z,e_i)$$

Now using (3.27) and $\mathrm{trace}(\varphi)=0$ in the above equation, we obtain (3.32) and (3.32) gives (3.33).

Theorem 12. The torsion tensor $\tilde{T}$ satisfies the following condition

$$\tilde{T}\left(\tilde{T}(X,Y)Z\right)+\tilde{T}\left(\tilde{T}(Y,Z)X\right)+\tilde{T}\left(\tilde{T}(Z,X)Y\right)=0 \qquad (3.34)$$

for all $X,Y,Z\in TM$

Proof: Using (2.4) and (3.1) we get (3.34).

REFERENCES

[1]. G. B. Folland, *Weyl manifolds, J. Di . Geometry 4(1970), 145-153*.

[2]. A. Friedmann, and J. A. Schouten, *Uber die geometrie der halbsymmetrischen ubertragung, Math. Zeitschr. 21(1924), 211-223.*

[3]. K.Yano, *On semi-symmetric metric connections, Rev. Roumaine Math. Pures Appl. 15(1970), 1579-1586. MR0275321(431078).*

[4]. N. S. Agashe, and M. R. Chae, *A semi-symmetric non-metric connection in a Riemannian manifold, Indian J. Pure Appl. Math. 23(1992), 399-409.*

[5]. O. C. Andonie, and D.Smaranda, *Certaines connexions semi-symetriques, Tensor 31(1977), 8-12.*

[6]. Y. Liang, On semi-symmetric recurrent-metric connection, *Tensor 55(1994), 107-112.*

[7]. J. Sengupta, U. C. De, and T. Q. Binh, *On a type of semi-symmetric non-metric connection on a Riemannian manifold, Indian J. Pure Appl. Math. 31(12)(2000), 1659-1670.*

[8]. S. Golab, *On semi-symmetric and quarter-symmetric linear connections, Tensor 29(1975), 249-254. MR 52 # 4156.*

[9]. R. S. Mishra, and, S. N. Pandey, *On quarter-symmetric metric F-connections, Tensor 34(1980), 1-7.*

[10]. S. C. Rastogi, *On quarter-symmetric metric connection, C. R. Acad. Bulg. Sci. 31(1978), no. 8, 811-814. MR 80f : 53015.*

[11]. S. C. Rastogi, *On quarter-symmetric metric connections, Tensor 44(1987), 133-141.*

[12]. K. Yano, and T. Imai, *Quarter-symmetric metric connections and their curvature tensors, Tensor 38(1982), 13-18.*

[13]. J.C. Marrero, *The local structure of trans-Sasakian manifolds, Ann. Mat. Pura Appl., 162(4) (1992), 77-86.*

[14]. K. Kenmotsu, *A class of almost contact Riemannian manifolds, Tohoku Math. J. 24(1972), no. 1, 93-103.*

[15]. M. M. Tripathi, *A new connection in Riemannian manifolds, Internat. Electr. J. Geom. 1(2008), no.1, 15-24.*

[16]. D.G. Prakasha , C.S. Bagewadi and N.S. Basavarajappa, *On Lorentzian β-Kenmotsu manifolds, Int.Journal of Math., 2(19)(2008), 919-927.*

[17]. S. Tanno , *The automorphism groups of almost contact Riemannian manifolds, Tohoku Math. J. 21 (1969), no. 1, 21-38.*

[18]. C.S. Bagewadi and E. Girish Kumar, *Note on Trans-Sasakian manifolds, Tensor.N.S., 65(1)(2004), 80-88.*

[19]. K.Yano, *Integral formulas in Riemannian geometry, Marcel Dekker Inc., 1970. MR0284950(442174).*

[20]. *J. A. Schouten , Ricci calculus, Springer, 1954*.

[21]. D.E. Blair, *Contact manifolds in Riemannian geometry, Lectures Notes in Mathematics, Springer-Verlag, Berlin, 509(1976), 146.*

[22]. D. E. Blair, *Riemannian geometry of contact and symplectic manifolds, Progress in Mathematics, 203. Birkhauser Boston, Inc., Boston, MA, 2002.*

[23]. R. S. Mishra , *Structures on a differentiable manifold and their applications, Chandrama Prakashan, Allahabad, India, 1984.*

[24]. H. A. Hayden, Subspaces of a space with torsion, Proc. London Math. Soc. 34(1932), 27-50.

[25]. D. Smaranda, *On projective transformations of symmetric connections with a recurrent projective tensor eld, Proc. Nat. Coll. on Geometry and Topology (Bu steni, 1981), 323-329, Univ. Bucare sti, Bucharest, 1983.*

[26]. J.C. Marrero and D. Chinea, *On trans-Sasakian manifolds, Proceedings of the XIVth Spanish-Portuguese Conference on Mathematics, Vol. I-III(Spanish)(Puerto de la Cruz, 1989), 655-659, Univ. La Laguna, La Laguna, 1990.*